

An Improved Unscented Kalman Filter Algorithm for Maneuvering Target Tracking

Jian Liu^a, Tian Yu^{b,*}

Shenyang Jianzhu University Faculty of Information and Control Engineering Shenyang, China

^ajeanliu@163.com, ^b42206009@qq.com

*Corresponding author

Keywords: Target tracking; Unscented kalman filter; Fading factor; Parameter adaptation; Noise estimator

Abstract: In order to improve the performance of the strong unscented Kalman filter (STUKF) for maneuvering target tracking and shorten the time required by the algorithm, a fast multi-fading STUKF algorithm with time-varying noise estimator is proposed. Considering the strategy of fading factor, the introduction position and reference method of the fading factor are improved. The filter gain and covariance matrix are adjusted adaptively. The process noise and observed noise with unknown statistical properties are estimated by adding a time-varying noise estimator to emphasize the importance of the adjacent data. The simulation results of various algorithms for the maneuvering target motion model are compared. The results show that the new filtering algorithm can track the maneuvering target better, improve the operating efficiency of UKF and the tracking accuracy. In the case of inaccurate noise statistics, the problem of filtering accuracy divergence is overcome, excessive calculation amount is avoided, and more efficient filtering performance is obtained.

1. Introduction

As an important optimal filtering theory, Kalman Filter (KF) is widely used in navigation, target tracking, communication and signal processing, fault diagnosis and detection. KF is calculated in recursive form and can estimate using limited measurement information containing noise. It has a small amount of data storage and has a lower system load than other detection methods. It is often used to predict the trend of dynamic systems. In the field of target tracking, the maneuvering target tracking technology based on KF and its extension algorithm is used in military and civilian equipment widely, which is of great significance. Tracking of the target's motion trajectory in dynamic systems reliably and accurately is one of the hotspots in this field. The follow-up of filtering technology provides more possibilities for the development of the target tracking field. In order to adapt to nonlinear system, algorithms such as extended Kalman filter (EKF), volumetric Kalman filter (CKF) and unscented Kalman filter (EKF) are proposed.

After continuous improvement, KF has the ability to adjust adaptively, which can better overcome the limitations of the system with many unknowns and insufficient information. Gao et al. use the equivalent weighting matrix and adaptive factor adjustment overcome the interference caused by the model error in the prediction [1]. Xia and Liu propose a filter composed of an interactive multi-model (IMM) combined with EKF, but this combination has large amount of calculation and the filtering process is complicated [2]. The interacting multiple model seventh-degree cubature Kalman filter (IMM-7th CKF) algorithm proposed by Ran and Qiao has higher filtering accuracy, but lacks the consideration of real-time noise changes, making the error between state quantity and observation. As time increases, it eventually leads to filter divergence [3]. The autoregressive prediction model is incorporated into KF for state estimation, and the real-time information of the innovation sequence is used to calculate the process noise covariance in Jin's literature [4]. The estimation accuracy is higher than the adaptive KF of the traditional discrete differential model.

Zhang et al. use an adaptive filter based on nonlinear CKF and covariance difference estimation to solve nonlinear problem [5], but as the number of filtering iterations increases, the covariance matrix may lose non-negative characterization and symmetry. Duan et al. use the adaptive square root cubature Kalman filter algorithm (SRCKF) to track the position of the model, which solves the numerical stability problem of CKF effectively and reduces the amount of calculation [6]. Huang et al. use two fading factors to adjust the error covariance matrix to form a strong robust CKF with good robustness [7]. Introducing the fading factor into the Kalman filter can overcome the shortcomings of KF's filtering accuracy when the system motion model is uncertain, and demonstrates the effectiveness of the strong tracking.

In this paper, a time-varying noise estimator is introduced into the strong tracking unscented Kalman filter (STUKF), and the position of the fading factor is adjusted. This fast multi-fade strong tracking unscented Kalman filter (F-MSTUKF) with noise estimation can perform characteristic statistics and real-time adaptive adjustment of noise with unknown variation. Adding the adjustment mechanism to the UKF's prediction and update steps avoids inaccurate or even diverging filtering. Comparing the above methods with the UKF and extended algorithms for nonlinear systems, it verifies that the improved STUKF method has better adaptive ability and tracking accuracy.

2. Fast Multi-Fade Strong Tracking Unscented Kalman Filter

Kalman filter was only available for linear systems originally, followed by EKF for nonlinear applications [8]. UKF is another filtering method applied to nonlinear systems. It eliminates the way of linearizing the system and uses the unscented transform to process the state prediction and update of the system. It has higher calculation accuracy [9]. The unscented transformation is transmitted by means of iterative method for obtaining the mean and covariance [10]. After acquiring the sigma sample points and calculating the corresponding weights, the new sample points is obtained by the unscented transformation according to the one-step prediction of the sample points, and the new sample points is brought in the observation equation to predict the observed value [11]. The observed values of the observations are weighted and summed to obtain the predicted mean and covariance. In the calculation process of UKF, the linearization approximation of Taylor expansion is not needed, but the mean and covariance are used to match the original statistical characteristics by using a series of sampling points, which overcomes the shortcomings of using linearization to cause stability error. Therefore, the target tracking based on the UKF has higher accuracy and requires less information on the system.

The filter gain is related to the bandwidth and convergence speed of the filter, which affects the weight of the innovation sequence in calculating the state and observation of the system. Adaptively adjusting the gain of the update phase according to the real-time state of the system is advantageous for improving the filtering accuracy. The strong tracking method uses the residual sequence to keep each other orthogonal to extract the information needed for system state prediction. The strong tracking implementation of the nonlinear system is to select an appropriate time-varying gain matrix to make the residual sequence orthogonal, it can be expressed as follows:

$$\begin{cases} E[(X_k - \hat{X}_k)(X_k - \hat{X}_k)^T] = \min, k = 0, 1, 2, \dots, \\ E[\varepsilon(k+1+j)\varepsilon^T(k+1)] = 0, j = 1, 2, 3, \dots \end{cases} \quad (1)$$

where $\varepsilon_k = Z(k) - \hat{Z}(k|k-1)$ is residual sequence.

When the system model and the real model are different or the system has a sudden change, the orthogonality of the residual will be affected. Equation.1 can maintain the robustness of the strong tracking process and ensure the orthogonal relationship. It is necessary to reduce the amount of calculation in the operation steps of STUKF. Therefore, an F-MSTUKF filtering algorithm with noise estimation is proposed. On the basis of changing the reference position of the fading factor to reduce the computational complexity, the multi-fade mechanism and noise estimation are added to the filtering algorithm to enhance the stability of the filtering.

2.1 Fast STUKF.

The idea of strong tracking introduces the fading factors into the state prediction and observation process of the system, adjusts the parameter adaptively in real time, reduces the influence of the old data pinning in the UKF, and it can improve the filtering performance effectively. According to the algorithm flow, the introduction of the fading factor is equivalent to repeating the calculation of sample points and covariance matrix in the measurement update, which increases the complexity of the algorithm. Reselecting the position of fading factors can reduce the extra calculation due to the factors, avoid the repeated execution of the same type of steps and reduce the algorithm complexity without affecting the accuracy of the algorithm. The position of the fading factors is adjusted to the observation measurement stage, and only changes the calculation of observation and covariance matrix. It affects the filter gain and state update indirectly [13]. The adjustments to the reference of fading factors is as follows:

$$\mathbf{Z}^{(i)}(k+1|k) - \hat{\mathbf{Z}}(k+1|k) = \sqrt{\lambda_{k+1}} \left[\mathbf{Z}_i'(k+1|k) - \hat{\mathbf{Z}}(k+1|k) \right], \quad (2)$$

$$\mathbf{P}_{z_k z_k} = \lambda(k+1) \sum_{i=1}^{2n} \omega^{(i)} \left[\mathbf{Z}_i'(k+1|k) - \hat{\mathbf{Z}}(k+1|k) \right] \cdot \left[\mathbf{Z}_i'(k+1|k) - \hat{\mathbf{Z}}(k+1|k) \right]^T + \mathbf{R}, \quad (3)$$

$$\mathbf{P}_{x_k z_k} = \lambda(k+1) \sum_{i=1}^{2n} \omega^{(i)} \left[\mathbf{X}^{(i)}(k+1|k) - \hat{\mathbf{X}}(k+1|k) \right] \cdot \left[\mathbf{Z}_i'(k+1|k) - \hat{\mathbf{Z}}(k+1|k) \right]^T, \quad (4)$$

where $\hat{\mathbf{Z}}$ is mean of the predicted values of the observation; λ stands for fading factor; \mathbf{P} stands for covariance matrix; \mathbf{R} is the covariance matrix of observed noise. The state covariance matrix of the update phase with fading factor is expressed as:

$$\mathbf{P}(k+1|k+1) = \lambda(k+1) \mathbf{P}'(k+1|k) - \mathbf{K}(k+1) \mathbf{P}_{z_k z_k} \mathbf{K}^T(k+1). \quad (5)$$

The above method can extract the residual information more completely and use the updated covariance matrix to influence the prediction at the next moment.

Since the time complexity of the step of calculating the sample point set is closely related to the complexity of the state vector dimension and the measurement function. For systems with higher state dimensions and more complex measurement functions, the optimization effect of time complexity is more obvious.

2.2 Multi-fade STUKF.

For nonlinear systems with multidimensional state variables, different state variables affected by the model deviation have different degrees, and the changes are random. If multiple fading factors are used for real-time adjustment, the estimation results can be more suitable for the operation of the real system [14]. Equation.6 is the construction of multiple fading factor matrix.

$$\boldsymbol{\lambda}_k = \text{diag} \left[\lambda_k^1, \lambda_k^2, \dots, \lambda_k^n \right], \quad (6)$$

where $\lambda_k^i \geq 1 (i=1, 2, \dots, n)$ is fading factor for each state variable.

The multiple fading factors quoted in EKF have the following form:

$$\lambda_k^i = \begin{cases} \alpha_i c_k, & \alpha_i c_k > 1 \\ 1, & \alpha_i c_k \leq 1 \end{cases}, \quad (7)$$

$$c_k = \frac{\text{tr}[\mathbf{N}(k)]}{\sum_{i=1}^n \alpha_i \mathbf{M}_k^{ii}}, \quad (8)$$

$$\mathbf{N}(k) = \mathbf{V}(k) - \mathbf{H}(k)\mathbf{Q}(k-1)\mathbf{H}^T(k) - \mathbf{R}, \quad (9)$$

$$\mathbf{M}(k) = \mathbf{H}\mathbf{F}(k|k-1)\mathbf{P}_{k-1}\mathbf{F}^T(k|k-1)\mathbf{H}^T, \quad (10)$$

where α_i is scale factor, its value depends on the error of the equation corresponding to the state variable. The larger the coefficient, the greater the degree of strong tracking. ρ is forgetting factor, its general value is 0.95.

$$\mathbf{V}(k) = \begin{cases} \varepsilon(1)\varepsilon^T(1) & k=1 \\ \frac{\rho\mathbf{V}(k-1) + \varepsilon(k)\varepsilon^T(k)}{1+\rho} & k>1 \end{cases}, \quad (11)$$

$$\mathbf{F}(k|k-1) = \frac{\partial f}{\partial \hat{\mathbf{X}}(k-1|k-1)}, \quad (12)$$

$$\mathbf{H}(k-1) = \frac{\partial h}{\partial \hat{\mathbf{X}}(k|k-1)}, \quad (13)$$

where $\mathbf{F}(k)$ is linearization matrix of state model; $\mathbf{H}(k)$ is linearization matrix of observation model.

UKF does not require the linearization process, and the calculation of fading factors in the UKF does not include the solution of partial derivative matrix [15]. According to the system process noise and the observed noise are irrelevant, and the noise covariance matrix is a positive definite symmetric matrix, Eq.14 can be derived.

$$\mathbf{H}_k = \left[\mathbf{P}_{xz}'(k+1|k) \right]^T \left[\mathbf{P}'(k+1|k) \right]^{-1}. \quad (14)$$

According to Eq.9 and Eq.14, we have

$$\mathbf{N}(k+1) = \mathbf{V}(k+1) - \left(\mathbf{P}_{xz}' \right)^T \left[\mathbf{P}'(k+1|k) \right]^{-1} \mathbf{Q}(k) \left[\mathbf{P}'(k+1|k) \right]^{-1} \mathbf{P}_{xz}' - \mathbf{R}. \quad (15)$$

Similarly, for (10) we have

$$\mathbf{M}(k+1) = \left(\mathbf{P}_{xz}' \right)^T \left[\mathbf{P}'(k+1|k) \right]^{-1} \left[\mathbf{P}'(k+1|k) - \mathbf{Q}(k) \right] \left[\mathbf{P}'(k+1|k) \right]^{-1} \mathbf{P}_{xz}'. \quad (16)$$

The deformed fading factors reference method combines the multiple fading mechanism and the optimization of the reference position, and substituting Eq.15 and Eq.16 into Eq.9 and Eq.10 can obtain the multi-fading factor we required for each iteration. Although multi-fading factors increase the amount of calculation, they have a good performance in improving the filtering accuracy.

3. Estimation of Noise Characteristics

For the estimation of actual system, when the characteristics of the noise are considered as the influencing factors, the estimation results will have higher accuracy. The assumption that the system's process noise and observed noise are considered as Gaussian white noise will lead to inaccurate observations in actual works and even occur filtering divergence. In order to make the nonlinear model used in the calculation closer to the real model of the system, the case where the mean of the noise is non-zero is taken into consideration [16]. For real-time noise, the estimation of noise characteristics should have different weights at different times. Introducing weight coefficients to emphasize the importance of adjacent data.

$$\begin{cases} \beta_i = \mu \cdot \beta_i \\ \sum_{i=0}^k \beta_i = 1 \end{cases} \rightarrow \beta_i = \frac{1-\mu}{1-\mu^{k+1}} \mu^{i-1}, \quad (17)$$

where μ is forgetting factor, and $0 < \mu < 1$ is satisfied.

Assuming in the weight coefficient, we have $\frac{1-\mu}{1-\mu^{k+1}} = a_k$, the calculation of the time-varying noise estimator is as follows:

$$\hat{\mathbf{q}}_k = (1-a_k)\hat{\mathbf{q}}_{k-1} + a_k \left[\hat{\mathbf{X}}_k - \sum_{i=1}^{2n} \omega_m^{(i)} \mathbf{X}^{(i)}(k|k-1) \right], \quad (18)$$

$$\hat{\mathbf{Q}}_k = (1-a_k)\hat{\mathbf{Q}}_{k-1} + a_k \left\{ \mathbf{K}_k \mathbf{E}_k \mathbf{E}_k^T \mathbf{K}_k^T + \mathbf{P}_k - \sum_{i=1}^{2n} \omega_c^{(i)} \left[\left(\mathbf{X}_{k|k-1}^{(i)} - \hat{\mathbf{X}}_{k|k-1} \right) \left(\mathbf{X}_{k|k-1}^{(i)} - \hat{\mathbf{X}}_{k|k-1} \right)^T \right] \right\}, \quad (19)$$

$$\hat{\mathbf{r}}_k = (1-a_k)\hat{\mathbf{r}}_k + a_k \left[\hat{\mathbf{Z}}_k - \sum_{i=0}^{2n} \omega_m^{(i)} \cdot \mathbf{Z}^{(i)}(k|k-1) \right], \quad (20)$$

$$\hat{\mathbf{R}}_k = (1-a_k)\hat{\mathbf{R}}_{k-1} + a_k \left\{ \mathbf{E}_k \mathbf{E}_k^T - \sum_{i=1}^{2n} \omega_c^{(i)} \left[\left(\mathbf{Z}_{k|k-1}^{(i)} - \hat{\mathbf{Z}}_{k|k-1} \right) \left(\mathbf{Z}_{k|k-1}^{(i)} - \hat{\mathbf{Z}}_{k|k-1} \right)^T \right] \right\}, \quad (21)$$

where \mathbf{q}_k and \mathbf{r}_k represent time-varying estimates of process noise and observed noise respectively; \mathbf{Q}_k and \mathbf{R}_k are the covariance matrices of the two kinds of noises.

Time-varying noise estimators with fading memory index weighting can forget the stale data at earlier times, emphasize the importance of adjacent data and reduce the impact of initial value bias. The use of noise estimation at each iteration makes the calculation process cumbersome, so the estimator calculation is introduced only when the current time is at a large absolute value of the estimated error, otherwise the estimation at the previous time is used. Fig. 1 illustrates the structure of the algorithm.

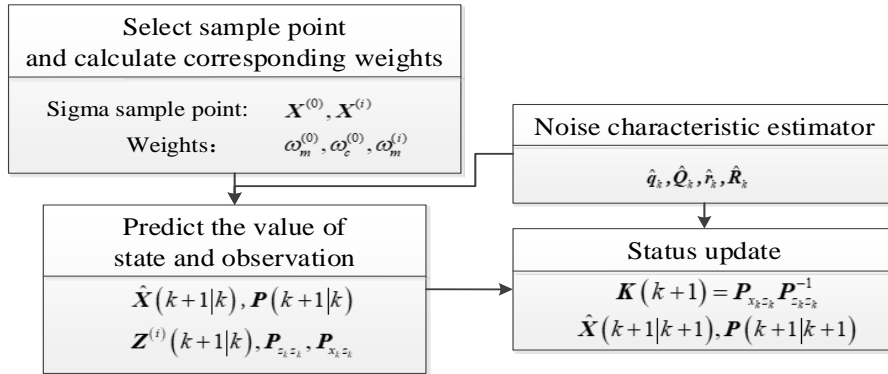


Fig. 1 Block diagram of F-MSTUKF

4. Simulation Results

In order to verify the improvement effect of the fading factors and noise estimator on the state and observation of nonlinear systems, compare the anti-jamming performance and running time of various algorithms, discuss the problem of maneuvering target tracking and analyze the tracking error of different filtering algorithms. In terms of the accuracy of the filtering, the root mean squared error is used as the evaluation index for the experimental results. The root mean squared error can directly measure the deviation between the observed experimental results and the real data, it is defined as follows:

$$R_{\text{MSE}} = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \hat{x}_i)^2}, \quad (22)$$

where x_i and \hat{x}_i are the real position and the estimated position at Monte Carlo simulation respectively. The smaller value of RMSE means the higher accuracy of the filtering algorithm.

4.1 Case 1: Maneuvering Target with Variable Acceleration.

Establish a maneuvering model of a moving target on a plane and use a radar to observe it. The scanning period of the radar is 2 seconds and the components of the horizontal and vertical axes are observed independently. In a certain period of time, the target travels along the longitudinal axis with the coordinate position of (2000, 10000) as the starting point. First, the target undergoes a slow turn of 90 degrees, the acceleration is $0.075 \text{ m} \cdot \text{s}^{-2}$. When the turn is over, the acceleration drops to zero and begins a quick turn with an angle of 90 degrees. The acceleration of the second turn is $0.3 \text{ m} \cdot \text{s}^{-2}$. The motive equation of the moving target is represented by

$$X(k+1) = \Phi X(k) + W(k). \quad (23)$$

Parameters in the equation are defined as follows:

$$X = [x \quad \dot{x} \quad y \quad \dot{y} \quad \ddot{x} \quad \ddot{y}]^T, \quad (24)$$

$$\Phi = \begin{bmatrix} 1 & T & 0 & 0 & \frac{T^2}{2} & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & T & 0 & \frac{T^2}{2} \\ 0 & 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (25)$$

where x and y are the coordinates of the target; \dot{x} and \dot{y} are velocities corresponding to the coordinates; \ddot{x} and \ddot{y} are accelerations; T is sampling period of the radar.

The statistical characteristic of the system noise is unknown. The statistical characteristic of the observed noise is known, and the observed noise covariance matrix is represented by

$$R_k = \text{diag}[100 \quad 0.01^2]. \quad (26)$$

Mean error of filtering is defined by

$$\overline{e_x(k)} = \frac{1}{M} \sum_{i=1}^M [x_i(k) - \hat{x}_i(k|k)] \quad (27)$$

where M is the number of Monte Carlo simulations; $k = 1, 2, \dots, N$, N is the number of samples.

After 50 times of Monte Carlo simulations, the tracking effect of the maneuvering target and the mean error curve for tracking are shown in Fig. 1 and Fig. 2. When the disturbance noise is added during the maneuvering target motion, the error curve has fluctuation. F-MSTUKF can still achieve the purpose of tracking before and after the two turns of the target. Compared with F-STUKF, it has better ability to cope with state mutations and smaller estimation error. It can be seen from the error curves in the two coordinate axes that the second turn has a larger non-error standard deviation than the first turn, and the mean error of the stable phase is small. By this kind of filter, the impact of maneuvering is small and it can track maneuvering targets better.

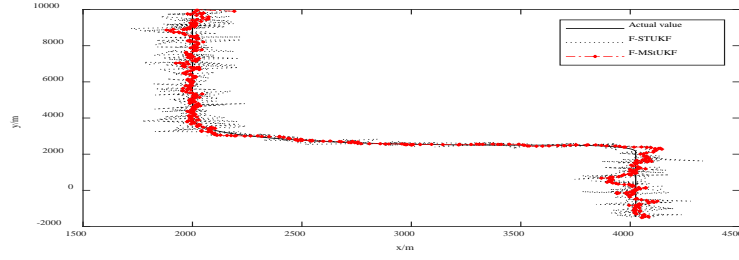


Fig. 2 Tracking results for maneuvering targets

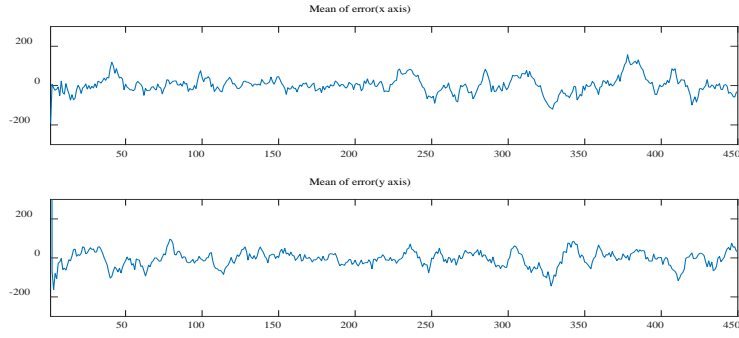


Fig. 3 Mean-error curve in the directions of x-axis and y-axis

The specific running time of the filtering algorithm can be used as the embodiment of the amount of calculation. Changing the position of gradual fading factors avoids the additional calculation of STUKF and shorten the time significantly, which makes the time cost of F-STUKF and F-MSTUKF close to UKF. The root mean square error of each algorithm is compared in Table 1. Compared with the UKF algorithm, the algorithm with strong tracking and fading factor has better accuracy. After fast strong tracking UKF simplified fading factors' reference, the accuracy of the filtering is affected. The combination of multiple fading factors and F-STUKF takes into account the two parameters of time cost and root mean square error. The tracking accuracy of F-MSTUKF is better than others, which is about 38.99% higher than F-STUKF, and only 9.45% increase in time.

Table 1 Performance of each algorithm in maneuvering target tracking

| Algorithm | Time (s) | RMSE |
|-----------|----------|-------|
| UKF | 1.233 | 1.932 |
| STUKF | 2.419 | 0.421 |
| F-STUKF | 1.302 | 0.536 |
| F-MSTUKF | 1.425 | 0.327 |

4.2 Case 2: Abrupt Change of Noise Characteristics.

The filter target is a variable speed target with two times of change in the observed noise characteristics. As shown in Fig. 3, the simulation time is $N=100$, and the mutation occurs at the time of 25 s and 80 s. In the process of filtering, if the original noise statistical model is still used to describe the noise characteristics, the error will be increased and the filter will be diverged. As shown in Fig. 4, when the time-varying noise estimator is added, the error waveform is smooth, the filtering accuracy is improved significantly and the filtering divergence is avoided. Therefore, when the system is affected by the change of statistical characteristics of noise, under the filtering mechanism with noise estimation can maintain a small estimation error and have better adaptive ability to variation of noise.

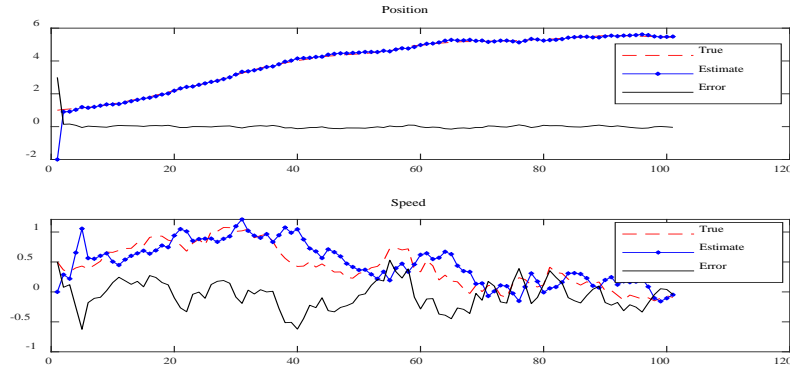


Fig. 4 Position and speed tracking without time-varying noise estimator

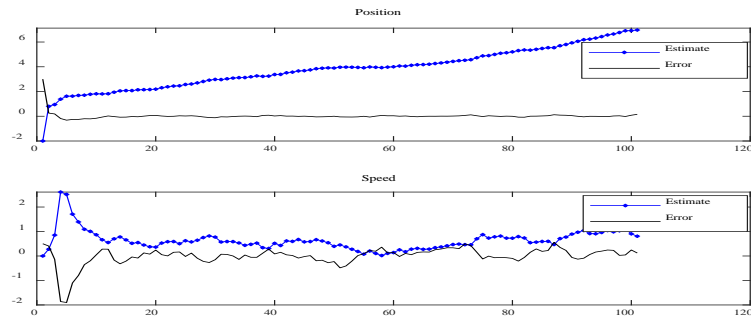


Fig. 5 Position and speed tracking with time-varying noise estimator

Table 2 compares the root mean square error before and after the noise estimation, the accuracy is improved about 15.79%. Since the calculation of the noise estimator is not referenced every time the filter is updated, it is only used when the absolute value of the estimated error is large, so the noise estimation has little effect on the time cost of the algorithm.

Table 2 Comparison of root mean square error of each algorithm

| Algorithm | Time (s) | RMSE |
|-------------------------------|----------|-------|
| UKF | 1.325 | 1.847 |
| F-MSTUKF | 1.433 | 0.304 |
| F-MSTUKF with noise estimator | 1.469 | 0.256 |

5. Conclusions

In order to improve the tracking accuracy and operational efficiency of STUKF, the reference method of multiple fading factors is improved, which directly affects the prediction of observation and covariance matrix. It adjusts the filtering gain in time. The algorithm also uses a time-varying noise estimator to estimate noise with unknown statistical properties, updates the noise adaptively to reduce the absolute deviation of the tracking results. It emphasizes the importance of the adjacent data. Compared with UKF and STUKF, it is shown that the proposed method can estimate the motion trajectory of the maneuvering target better. The F-MSTUKF improves the accuracy and stability of the filter tracking without increasing the amount of calculation. The combination of tracking technology and Kalman filtering is one of the important development directions in the field of automatic control. With the continuous improvement of filtering technology and the technical follow-up of control theory, its application field will be more extensive.

Acknowledgment

This work is supported by the scientific research projects in National Natural Science Foundation of China (11704263), Liaoning Province Natural Science Foundation (201602616) and Liaoning Province Department of Education Scientific Research Project (2015443).

References

- [1] Z.H. Gao, D.J. Mu and S.S. Gao: Robust adaptive filter allowing systematic model errors for transfer alignment. *Aerospace Science and Technology*, Vol. 59 (2016), p.32-40. J. Clerk Maxwell, *A Treatise on Electricity and Magnetism*, 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68–73.
- [2] X.H. Xia, M. Liu: Battery state estimation based on interactive multi-model Kalman filter. *Information and Control*, Vol. 46 (2017) No.5, p.519-524. (In Chinese)
- [3] N. Ran, X. Qiao: Interactive multi-model seventh-order volume Kalman filter algorithm. *Journal of Electronic Measurement and Instrumentation*, Vol. 32 (2018) No.6, p.167-172. (In Chinese)
- [4] B. Jin, J. Guo and D.J He: Adaptive Kalman filtering based on optimal autoregressive predictive model. *GPS Solutions*, Vol. 21 (2017) No.2, p.307-317. (In Chinese)
- [5] Y. Zhang, J.G. Wang and Q. Sun: Adaptive cubature Kalman filter based on the variance-covariance components estimation. *The Journal of Global Positioning Systems*, Vol. 15 (2017) No.1, p.1-9.
- [6] J.M. Duan, H. Shi and D. Liu: Square root cubature Kalman filter-Kalman filter algorithm for intelligent vehicle position estimate. *Procedia Engineering*, Vol. 137 (2016), p.267-276.
- [7] W. Huang, H.S. Xie and C. Shen: A robust strong tracking cubature Kalman filter for spacecraft attitude estimation with quaternion constraint. *Acta Astronautica*, Vol. 121 (2016), p.153-163.
- [8] L.C. Shi, Y.L. An and B.H. Su: An improved background subtraction algorithm based on Kalman filtering[J]. *Laser & Optoelectronics Progress*. *Laser & Optoelectronics Progress*, Vol. 55 (2018) No.8, p.081003. (In Chinese)
- [9] X.X. Yang, L.W. Deng and H.F. Chen: Video target tracking using strong tracking UKF. *Electronic Measurement Technology*, Vol. 39 (2016) No.10, p.95-99.
- [10] P. Jiang, H.H. Song and X.M. Wang: Target tracking algorithm for sensor networks based on dynamic spanning tree and improved unscented Kalman filter. *Chinese Journal of Scientific Instrument*, Vol. 36 (2015) No.2, p.415-421. (In Chinese)
- [11] S. Zhan, S.M. Feng: Comparative study on performance of extended Kalman filter and unscented Kalman filter. *Information & Communications*, Vol. 5 (2018), p.35-36. (In Chinese)
- [12] R. Wang, C.S. Zhao and Z.H. Xia, Research on attenuation memory algorithm of unscented Kalman filter. *Bulletin of Surveying and Mapping*, Vol. 0 (2015) No.12, p.20-22. (In Chinese)
- [13] X.G. Zhang, Y. Zhang and Y.N. Wang: Covariance tracking based on forgetting factor and Kalman filter. *Acta Optica Sinica*, Vol. 30 (2010) No.8, p.2317-2323. (In Chinese)
- [14] X.S. Gan, W.M. Gao and Z. Dai: Research on WNN soft fault diagnosis for analog circuit based on adaptive UKF algorithm. *Applied Soft Computing*, Vol. 50 (2017), p.252-259.
- [15] G.Yu. Kulikov, M.V. Kulikova: Accurate cubature and extended Kalman filtering methods for estimating continuous-time nonlinear stochastic systems with discrete measurements. *Applied Numerical Mathematics*, Vol. 111 (2017), p.260-275.
- [16] C. Liu, S.C. Yang and L.D. Wang: Target tracking algorithm based on adaptive strong tracking CQKF. *Journal of Beijing University of Aeronautics and Astronautics*, Vol. 44 (2018) No.5, p.982-990. (In Chinese)